

Insights and possible resolution to the information loss paradox via the tunneling picture

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Abstract

This paper investigates the information loss paradox in the WKB/tunneling picture of Hawking radiation. In the tunneling picture one can obtain the tunneling amplitude to all orders in \hbar . However all terms beyond the lowest, semi-classical term involve unknown constants. Despite this we find that one can still arrive at interesting restrictions on Hawking radiation to all orders in \hbar : (i) Taking into account only quantum corrections the spectrum remains thermal to all orders. Thus quantum corrections by themselves will not resolve the information loss paradox. (ii) The first quantum correction give a temperature for the radiation which goes to zero as the mass of the black hole goes to zero. Including higher order corrections changes this nice result of the first order corrections. (iii) Finally we show that by taking both quantum corrections and back reaction into account it is possible under specific conditions to solve the information paradox by having the black hole evaporate completely with the information carried away by the correlations of the outgoing radiation.

Keywords: Information paradox, tunneling beyond semi-classical approximation, blackbody spectrum

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I. INTRODUCTION

There are many derivations of Hawking radiation in literature. Recently this phenomenon has been studied using a variant of the WKB/tunneling approach [1]. This approach provides a simple and physically intuitive picture. The tunneling method has two versions: (i) the null geodesic method [1]; (ii) the Hamilton-Jacobi method [2]. The original work on the tunneling method was unable to obtain the thermal nature of the Hawking radiation. This shortcoming was recently solved by Banerjee and Majhi [3] who obtained the thermal spectrum for Hawking radiation using the tunneling picture. The thermal nature of Hawking radiation leads to the information loss paradox: since the radiation is thermal there are no correlations between the emitted field quanta and so one loses information about the nature of the matter that originally formed the black hole. More technically, the complete evaporation of a black hole, whereby a pure quantum state evolves into a thermal state, would violate the quantum mechanical unitarity. A recent review of the information paradox can be found in reference [4].

There have been various proposals for resolving this information loss paradox which usually involve the idea that Hawking radiation is modified when one goes beyond the approximations used in the various methods of deriving it. One possibility is that higher order corrections in \hbar may make the spectrum non-thermal. For example, in the case of charged particle pair creation in a uniform electric field – the Schwinger effect [5] which can be thought of as a non-GR, field theory version of Unruh/Hawking radiation [6] – the tunneling amplitude can be calculated exactly to all orders in \hbar and it is non-thermal. Here we show that the same is not true for Hawking radiation – using the tunneling formalism it can be shown that to all orders of \hbar the radiation spectrum is thermal. Thus, even taking the full quantum theory into account does not resolve the information paradox [21].

Quantum tunneling with back reaction gives the relation between the tunneling rate and entropy change of black hole [1]. This relation provides an alternative approach to the information loss paradox [7, 8]. However, the considerations are confined to the semi-classical approximation. The approach to the information loss paradox given here involves both quantum corrections to all orders and back reaction. For this approach we look at the information loss paradox in terms of entropy. The original black hole has some entropy S which is proportional to the surface area, A , of the black hole [9]. If the black hole evaporates

away completely into thermal radiation then the total entropy after the evaporation will be greater than that before. From the microscopic view of entropy this indicates that the number of microstates is greater after evaporation as compared to before evaporation. This increase in the number of states indicates that the evolution is non-unitary. In order to have unitary evolution the entropy before should be equal to the entropy after. In this article we will show that it is possible under specific conditions for the entropy before, as determined by the surface area of the black hole, to be made equal to the entropy of the outgoing radiation after the black hole has completely evaporated. The entropy of the outgoing radiation will be calculated from the non-trivial correlations between emitted field quanta which arise due to higher order quantum corrections plus back reaction.

II. BLACKBODY SPECTRUM BEYOND THE SEMI-CLASSICAL APPROXIMATION

In this section we briefly review the salient results from references [10, 11] where a formalism was developed to investigate quantum corrections *to all orders* in \hbar in the tunneling method.

We begin by considering a general class of static, spherically symmetric space-time

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2. \quad (1)$$

The horizon of the black hole $r = r_H$ is given by $f(r_H) = 0$. In this space-time, a massless scalar particle obeys the Klein-Gordon equation

$$-\frac{\hbar^2}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu)\phi = 0. \quad (2)$$

In the tunneling framework, the tunneling particle is considered as a spherical shell so that the trajectory of the tunneling process is radial. Therefore only the (r, t) sector of the metric (1) is important and thus tunneling of a particle from a black hole can be considered as a two-dimensional quantum process in the (r, t) plane. For a two dimensional theory, the standard WKB ansatz for the wave function ϕ can be expressed as

$$\phi(r, t) = \exp\left[\frac{i}{\hbar}I(r, t)\right] \quad (3)$$

where $I(r, t)$ is the one particle action which can be expanded in powers of \hbar as

$$I(r, t) = I_0(r, t) + \sum_j \hbar^j I_j(r, t). \quad (4)$$

Here $I_0(r, t)$ is the semi-classical action and the others are quantum corrections. As shown in [10, 11], $I_i(r, t)$ are proportional to $I_0(r, t)$, so that (4) can be written as

$$I(r, t) = \left(1 + \sum_j \gamma_j \hbar^j \right) I_0(r, t) \quad (5)$$

with γ_j being unknown proportionality constants. We are using units with $G = c = k_B = 1$ so that \hbar has units of length square – the Planck length l_P . The solution of the semi-classical action I_0 has the form

$$I_0(r, t) = \omega(t \pm r_*), \quad r_* = \int \frac{dr}{f(r)}. \quad (6)$$

The $- (+)$ sign refers to outgoing (ingoing) trajectories. For the high order correction terms in equation (5), the undetermined proportionality coefficients γ_j have dimension \hbar^{-j} . For concreteness we consider a Schwarzschild black hole whose only macroscopic parameter can be defined in terms of the radius of the horizon r_H . Performing some elementary dimensional analysis – requiring that $\gamma_j \hbar^j$ be dimensionless and remembering that in our units \hbar has units of l_P^2 – one can show the coefficients γ_j have dimension r_H^{-2j} . This is expressed as

$$\gamma_j = \frac{\alpha_j}{r_H^{2j}} \quad (7)$$

with α_j dimensionless constants. Thus the solution for the scalar field $\phi(r, t)$ is,

$$\phi(r, t) = \exp \left[\frac{i}{\hbar} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \omega(t \pm r_*) \right]. \quad (8)$$

If $(t \pm r_*)$ has an imaginary part then $\phi(r, t)$ will have an exponentially decreasing part from which the tunneling amplitude, the temperature and the spectrum of the radiation can be found. From equation (6) and performing a contour integration it has been shown [12] that r_* does have an imaginary part but gives twice the correct Hawking temperature. In [13] it was shown that in general there is also an imaginary contribution coming from the time part of $(t \pm r_*)$. This is seen by introducing the Kruskal time (T) and space (X) coordinates inside and outside the horizon

$$\begin{aligned} T_{\text{in}} &= \sqrt{1 - 2\kappa r} e^{\kappa r} \cosh(\kappa t_{\text{in}}), & X_{\text{in}} &= \sqrt{1 - 2\kappa r} e^{\kappa r} \sinh(\kappa t_{\text{in}}), \\ T_{\text{out}} &= \sqrt{2\kappa r - 1} e^{\kappa r} \sinh(\kappa t_{\text{out}}), & X_{\text{out}} &= \sqrt{2\kappa r - 1} e^{\kappa r} \cosh(\kappa t_{\text{out}}), \end{aligned} \quad (9)$$

where $\kappa = \frac{f'(r_H)}{2}$ is the surface gravity of the black hole. The inside ($T_{\text{in}}, X_{\text{in}}$) and outside ($T_{\text{out}}, X_{\text{out}}$) coordinates can be connected by the following transformation

$$t_{\text{in}} = t_{\text{out}} + \frac{i\pi}{2\kappa}. \quad (10)$$

Thus, on crossing the horizon the time part picks up an imaginary part. This temporal contribution plays an important role in determining the correct Hawking temperature for black holes in the tunneling method [13]. Under this transformation the inside and outside modes of the scalar field (8) are connected by

$$\phi_{\text{in}}^{\text{R}} = \exp \left[-\frac{\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right] \phi_{\text{out}}^{\text{R}}, \quad \phi_{\text{in}}^{\text{L}} = \phi_{\text{out}}^{\text{L}}. \quad (11)$$

Here in, out means modes inside/outside the horizon and R, L indicate modes moving to the right/left (away/toward $r = 0$). We now review the parts of the derivation of the spectrum [3] that will be important to showing that to all orders in \hbar the spectrum still remains thermal. Consider the physical state with n non-interacting virtual pairs created inside the black hole. When observed from outside, this state is given by

$$|\Psi\rangle = N \sum_n |n_{\text{in}}^{\text{L}}\rangle \otimes |n_{\text{in}}^{\text{R}}\rangle = N \sum_n \exp \left[-\frac{n\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right] |n_{\text{out}}^{\text{L}}\rangle \otimes |n_{\text{out}}^{\text{R}}\rangle \quad (12)$$

where N is the normalization constant and $|n^{\text{L(R)}}\rangle$ represents n number of the left(right)-moving mode. In the above expression, we have used the relations (11). N can be determined from the normalization condition, which leads to

$$N = \sqrt{1 \mp \exp \left[-\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right]} \quad (13)$$

where the $- (+)$ sign is for bosons (fermions). One can define the density matrix operator for the system as

$$\begin{aligned} \rho_{\text{boson, fermion}} &= |\Psi\rangle_{(\text{boson, fermion})} \langle\Psi|_{(\text{boson, fermion})} \\ &= \left[1 \mp e^{-\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)} \right] \sum_{m,n} e^{-\frac{(n+m)\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)} |n_{\text{out}}^{\text{L}}\rangle \otimes |n_{\text{out}}^{\text{R}}\rangle \langle m_{\text{out}}^{\text{R}}| \otimes \langle m_{\text{out}}^{\text{L}}|. \end{aligned} \quad (14)$$

It should be stressed that the left-moving modes are going in the direction of $r = 0$ and thus they are not observed by an outside observer. Therefore, one can trace out the left-moving

modes, giving the density matrix operator for the right-moving modes as

$$\rho_{\text{boson,fermion}}^{\text{R}} = \left(1 \mp e^{-\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)} \right) \sum_n e^{-\frac{2n\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)} |n_{\text{out}}^{\text{R}}\rangle \langle n_{\text{out}}^{\text{R}}|. \quad (15)$$

From this one can immediately get the average number of particles detected at asymptotic infinity

$$\langle n \rangle_{(\text{boson,fermion})} = \text{trace} \left(\hat{n} \hat{\rho}_{(\text{boson,fermion})}^{\text{R}} \right) = \frac{1}{e^{\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)} \mp 1}. \quad (16)$$

This result is the Bose-Einstein distribution (for $-$ sign) or Fermi-Dirac distribution (for $+$ sign). Both these distributions correspond to a blackbody spectrum with a corrected Hawking temperature given by

$$T = \frac{\hbar\kappa}{2\pi} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right)^{-1}. \quad (17)$$

In this expression $\frac{\hbar\kappa}{2\pi}$ is the semi-classical Hawking temperature, T_H , and the other terms are higher order quantum corrections.

We now give an alternative derivation of the quantum corrected temperature in terms of the tunneling rate, Γ , since this will be important for the next section when we make a connection between Γ and the change in entropy. The tunneling rate can be written as $\Gamma = \frac{P^{\text{R}}}{P^{\text{L}}}$, where $P^{\text{R,L}}$ are the probabilities of the right/left-moving modes to cross the horizon. From equation (11) one finds

$$P^{\text{R}} = |\phi_{\text{in}}^{\text{R}}|^2 = \left| \exp \left[-\frac{\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right] \phi_{\text{out}}^{\text{R}} \right|^2 = \exp \left[-\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right], \quad (18)$$

and

$$P^{\text{L}} = |\phi_{\text{in}}^{\text{L}}|^2 = |\phi_{\text{out}}^{\text{L}}|^2 = 1. \quad (19)$$

This result for P^{L} shows that with probability 1 left-moving modes cross the horizon, while the result for P^{R} gives the probability that modes tunneling out from behind the horizon. Combining these results and equating the tunneling rate, Γ , with the Boltzmann distribution, i.e. $e^{-\frac{\omega}{T}}$,

$$\Gamma = \frac{P^{\text{R}}}{P^{\text{L}}} = \exp \left[-\frac{2\pi\omega}{\hbar\kappa} \left(1 + \sum_j \frac{\alpha_j \hbar^j}{r_H^{2j}} \right) \right] = e^{-\frac{\omega}{T}}, \quad (20)$$

one can again recover the result for the corrected temperature (17).

Although the coefficients, α_j , are undetermined there are still two important conclusions one can draw from the results in equations (16) and (17): (i) The spectrum is still thermal to all orders of \hbar . Thus, the information loss paradox is not resolved by higher order corrections in \hbar to any order. Note the difference with the Schwinger effect where it is possible to calculate the decay amplitude to all orders and one finds there that the spectrum for charge creation in a strong electric field is non-thermal. (ii) In the lowest order one finds that the temperature diverges as the black hole evaporates, i.e. $T \rightarrow \infty$ as $M \rightarrow 0$. Taking into account the quantum corrections (and using $\kappa = \frac{1}{4M}$ and $r_H = 2M$) the temperature in equation (17) becomes

$$T = \frac{\hbar}{8\pi \left(M + \frac{\alpha_1 \hbar}{4M} + \frac{\alpha_2 \hbar^2}{16M^3} + \dots \right)}.$$

Taking into account the correction to the temperature up to the first quantum correction (i.e. $\frac{\alpha_1 \hbar}{4M}$ term) one sees that the denominator goes to ∞ as $M \rightarrow 0$, so now $T \rightarrow 0$ as $M \rightarrow 0$. Thus the generic first order quantum correction appears to modify the lowest order divergence of the temperature as $M \rightarrow 0$. (In the case when $\alpha_1 < 0$ there is still some finite M – i.e. $M = \sqrt{-\alpha_1 \hbar/4}$ – where T diverges as can be seen from figure 2(b)). Including higher order quantum corrections would appear to also always lead to a zero temperature as $M \rightarrow 0$. However there are cases – such as if the α_i 's alternate in sign – when this result of a zero temperature in the limit $M \rightarrow 0$ is not always the result. In the next section when we discuss the entropy we will find that in order to have a well behaved entropy as $M \rightarrow 0$ we need to have α_i 's which alternate in sign. Thus the apparent nice behavior of the temperature as $M \rightarrow 0$ is not guaranteed. This point will be discussed further in the next section.

III. BACK REACTION AND ENTROPY CONSERVATION

In the last section we found that quantum corrections to any order do not resolve the information paradox but they do tend to make the temperature go to zero, rather than diverging, as the mass of the black hole is evaporated away to zero. We now turn to back reaction effects. The tunneling approach allows one to take into account the effect of the emission of the Hawking radiation of the horizon in a simple manner using energy conservation [1]. Here we combine the back reaction effects along with the \hbar corrections

from the previous section to show that it is *possible* under specific conditions to resolve the information paradox.

One can find an expression to all orders in \hbar for the entropy by integrating the first law of thermodynamics, namely $dM = TdS$ where M is the initial mass of the black hole, T is temperature and S is entropy. Thus, $S = \int \frac{dM}{T}$ which upon substituting the expression for the modified temperature from equation (17) gives the modified entropy as a function of M

$$S(M) = \frac{4\pi}{\hbar} M^2 + \pi\alpha_1 \ln\left(\frac{M^2}{\hbar}\right) - \pi \sum_{j=1}^{\infty} \frac{\alpha_{j+1}}{4^j j} \left(\frac{\hbar}{M^2}\right)^j. \quad (21)$$

Now in general as $M \rightarrow 0$ this corrected expression for S diverges unless all $\alpha_j = 0$ in which case $S \rightarrow 0$. This has led to the idea that the black hole evaporation will stop with a black hole “remnant” of some undetermined mass m_R [16]. Here we would like to point out another possibility. We impose the condition that the unknown α_j ’s take values that allow $S \rightarrow 0$ as $M \rightarrow 0$. This condition can be met by setting

$$\alpha_{j+1} = \alpha_1 (-4)^j \quad \text{for } j = 1, 2, 3, \dots \quad (22)$$

Employing this condition, the sum in equation (21), i.e. the third term, becomes equal to $+\alpha_1 \pi \ln(1 + \hbar/M^2)$ and the entropy now reads

$$S(M) = \frac{4\pi}{\hbar} M^2 + \pi\alpha_1 \ln\left(1 + \frac{M^2}{\hbar}\right). \quad (23)$$

It is easily seen that for $M \rightarrow 0$ the entropy tends to zero, i.e. $S \rightarrow 0$. The behavior of entropy $S(M)$ versus black hole mass M have been shown in figure 1. Strictly the identification of the sum in equation (21) with $\alpha_1 \pi \ln(1 + \hbar/M^2)$ is only valid for $\sqrt{\hbar} < M$, i.e. when the mass, M , is larger than the Planck mass. However we can use analytic continuation to define the sum via $\alpha_1 \pi \ln(1 + \hbar/M^2)$ even for $\sqrt{\hbar} > M$. This is analogous to the trick in String Theory [17] where the sum $\sum_{j=1}^{\infty} j$ is defined as $\zeta(-1) = -1/12$ using analytic continuation of the zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$. Although the choice of α_j ’s in equation (22) is done by hand, one can view this in the following way: When a consistent theory of quantum gravity is found it should give the full expression for $S(M)$. For example, Loop Quantum Gravity gives $\alpha_1 = -1/2$ [18]. By setting the physical condition that $S(M = 0) \rightarrow 0$ we find what the α_j ’s from a complete theory of quantum gravity should be. This is in some sense “looking at the back of the book for the answer”. One can ask how unique is the choice in equation (22)? Are there other choices which would yield

$S(M = 0) \rightarrow 0$? As far as we have been able to determine there are no other choices of α_j 's that would give $S(M = 0) \rightarrow 0$, but for the time being we can not provide a formal proof. In regard to the temperature the choice of α_j 's in equation (22) leads to a geometric series and T takes on the following form

$$T = \frac{\hbar}{8\pi \left(M + \frac{\alpha_1 \hbar M}{4(\hbar + M^2)} \right)}. \quad (24)$$

If α_1 is independent of M then as $M \rightarrow 0$ one has $T \rightarrow \infty$. By allowing α_1 to depend on M one can have both a finite entropy and temperature as $M \rightarrow 0$. One example of this is the choice

$$\alpha_1 = K \frac{\hbar + M^2}{M^2 + \sqrt{\hbar}M} \quad (25)$$

where K is a dimensionless constant. The particular combination of M and \hbar is required to make α_1 dimensionless. This choice of α_1 gives temperature and entropy of the form

$$T = \frac{\hbar}{8\pi \left(M + \frac{K\hbar}{4(\sqrt{\hbar} + M)} \right)}, \quad S = \frac{4\pi M^2}{\hbar} + 2\pi K \ln \left(1 + \frac{M}{\sqrt{\hbar}} \right). \quad (26)$$

This expression for the entropy is essentially the same as in (23) with $\alpha_1 \rightarrow 2K$ and $M^2/\hbar \rightarrow M/\sqrt{\hbar}$ inside the logarithm. For the temperature given above we find $T \rightarrow \sqrt{\hbar}/2\pi K$ as $M \rightarrow 0$. In the following we will simply take α_1 to be independent of M and \hbar since to address the information loss puzzle we only need the entropy to take the form given in (23) or (26) which happens whether we pick α_1 as in (25) or let α_1 be independent of M and \hbar . In figures (1) and (2) we show the behavior of the entropy and temperature for both positive and negative α_1 . The dashed line is the lowest order result; the dotted line is the lowest order plus first order quantum correction; the dashed line is the all orders results for our particular choice of α_i 's.

We now want to show that the original entropy of the black hole given in equation (23) can be completely carried away by the emitted radiation when specific conditions are satisfied. Thus, the entropy is conserved which implies a unitary evolution and thus under specific conditions gives a possible resolution to the information loss paradox. To begin we look at the connection between the tunneling rate (20) and the change in entropy as given in [1]

$$\Gamma = e^{\Delta S} \quad (27)$$

where $\Delta S = S(M - \omega) - S(M)$ is the entropy change during the tunneling process when the black hole mass goes from M to $M - \omega$ due to the emission of a field quantum of energy

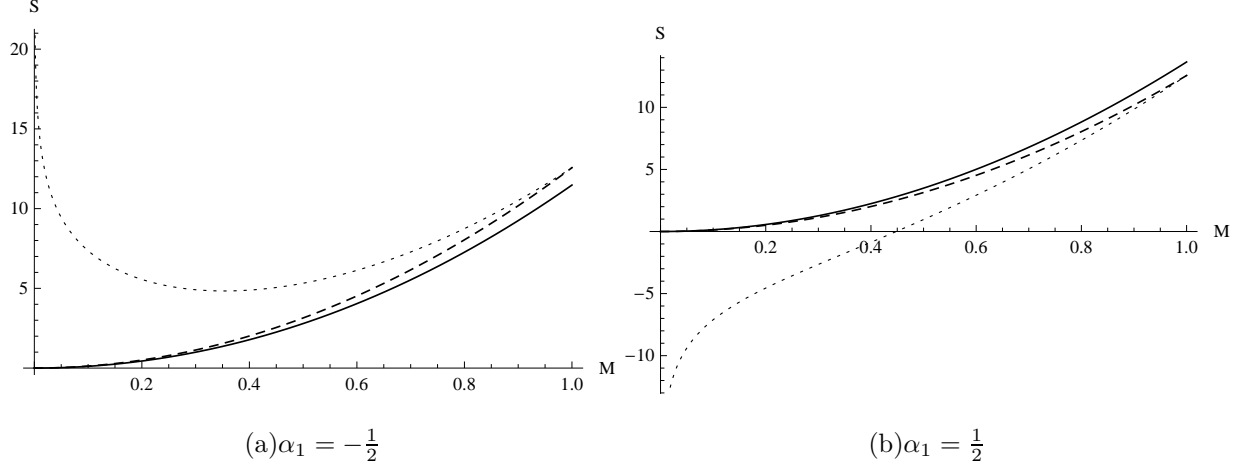


FIG. 1: The entropy S versus the black hole mass M . The three curves correspond to the lowest order entropy (dashed curve), the entropy to the first order correction (dotted curve), and the entropy with quantum corrections to all orders (solid curve) respectively.

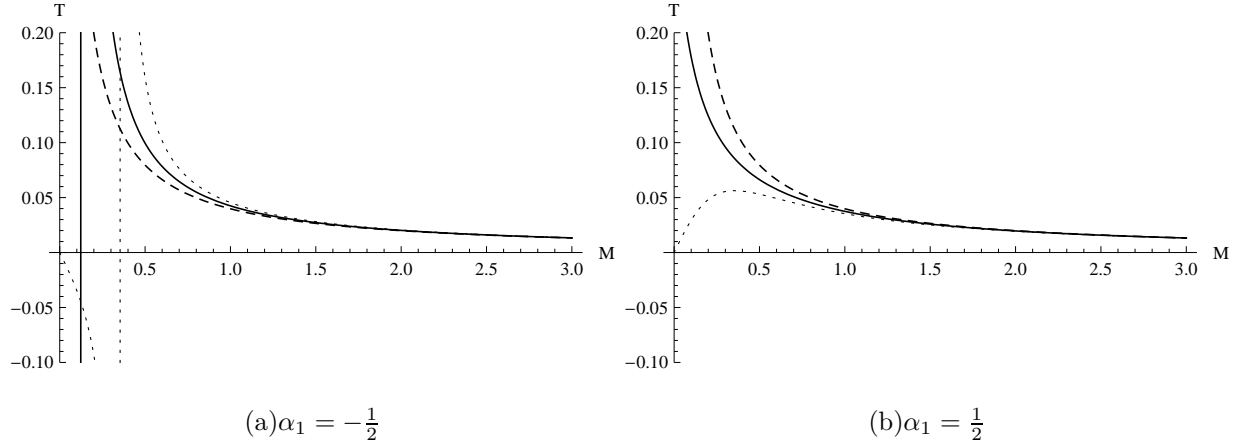


FIG. 2: The Hawking temperature T versus the black hole mass M . The three curves are corresponding to the lowest order temperature (dashed curve), the temperature to the first order correction (dotted curve), and the temperature with quantum corrections to all orders (solid curve) respectively.

ω . Using (23) one finds the following expression for ΔS

$$\Delta S = -\frac{8\pi}{\hbar}\omega \left(M - \frac{\omega}{2}\right) + \pi\alpha_1 \ln \left[\frac{1 + (M - \omega)^2/\hbar}{1 + M^2/\hbar} \right]. \quad (28)$$

Now combining equations (27) and (28), the corrected tunneling rate takes the form

$$\Gamma(M; \omega) = \left(\frac{1 + (M - \omega)^2/\hbar}{1 + M^2/\hbar} \right)^{\pi\alpha_1} \exp \left[-\frac{8\pi}{\hbar}\omega \left(M - \frac{\omega}{2}\right) \right]. \quad (29)$$

The term $\exp \left[-\frac{8\pi}{\hbar} \omega \left(M - \frac{\omega}{2} \right) \right]$ represents the result of back reaction on the tunneling rate while the term to the power $\pi\alpha_1$ represents the quantum corrections to all orders. Even in the classical limit, i.e. setting $\pi\alpha_1 = 0$, there is a deviation from a thermal spectrum due to the ω^2 term in the exponent.

We now find the connection between the tunneling rate (29) and the entropy of the emitted radiation, S_{rad} . Assuming that the black hole mass is completely radiated away we have the relationship $M = \omega_1 + \omega_2 + \dots + \omega_n = \sum_{j=1}^n \omega_j$ between the mass of the black hole and the energy, ω_j , of the emitted field quanta. The probability for this radiation to occur is given by the following product of Γ 's which are defined in equation (29) [14]

$$P_{rad} = \Gamma(M; \omega_1) \times \Gamma(M - \omega_1; \omega_2) \times \dots \times \Gamma \left(M - \sum_{j=1}^{n-1} \omega_j; \omega_n \right) \quad (30)$$

where the probability of emission of the individual field quanta of energy ω_j is given by

$$\begin{aligned} \Gamma(M; \omega_1) &= \left(\frac{1 + (M - \omega_1)^2/\hbar}{1 + M^2/\hbar} \right)^{\pi\alpha_1} \exp \left[-\frac{8\pi}{\hbar} \omega_1 \left(M - \frac{\omega_1}{2} \right) \right], \\ \Gamma(M - \omega_1; \omega_2) &= \left(\frac{1 + (M - \omega_1 - \omega_2)^2/\hbar}{1 + (M - \omega_1)^2/\hbar} \right)^{\pi\alpha_1} \exp \left[-\frac{8\pi}{\hbar} \omega_2 \left(M - \omega_1 - \frac{\omega_2}{2} \right) \right], \\ &\dots, \end{aligned} \quad (31)$$

$$\begin{aligned} \Gamma \left(M - \sum_{j=1}^{n-1} \omega_j; \omega_n \right) &= \left(\frac{1 + (M - \sum_{j=1}^{n-1} \omega_j - \omega_n)^2/\hbar}{1 + (M - \sum_{j=1}^{n-1} \omega_j)^2/\hbar} \right)^{\pi\alpha_1} \exp \left[-\frac{8\pi}{\hbar} \omega_n \left(M - \sum_{j=1}^{n-1} \omega_j - \frac{\omega_n}{2} \right) \right] \\ &= \left(\frac{1}{1 + (M - \sum_{j=1}^{n-1} \omega_j)^2/\hbar} \right)^{\pi\alpha_1} \exp(-4\pi\omega_n^2/\hbar). \end{aligned}$$

The Γ 's of the form $\Gamma(M - \omega_1 - \omega_2 - \dots - \omega_{j-1}; \omega_j)$ represent the probability for the emission of a field quantum of energy ω_j with the condition that first field quanta of energy $\omega_1 + \omega_2 + \dots + \omega_{j-1}$ have been emitted.

Now using (31) in equation (30) we find the total probability for the radiation process described above

$$P_{rad} = \left(\frac{1}{1 + M^2/\hbar} \right)^{\pi\alpha_1} \exp(-4\pi M^2/\hbar). \quad (32)$$

The black hole mass could also have been radiated away by a different sequence of field quanta energies e.g. first ω_2 is emitted and then other field quanta are emitted in the following order $\omega_2 + \omega_1 + \dots + \omega_{n-1} + \omega_n$. Assuming each of these different processes has

the same probability one can count the number of microstates, Ω , for the above process as $\Omega = 1/P_{rad}$. Then using the Boltzmann definition of entropy as the natural logarithm of the number of microstates we have

$$S_{rad} = \ln(\Omega) = \ln(1/P_{rad}) = \frac{4\pi}{\hbar} M^2 + \pi\alpha_1 \ln \left(1 + \frac{M^2}{\hbar} \right) . \quad (33)$$

This entropy of the emitted radiation (33) is identical to the original entropy of the black hole (23), thus entropy is conserved between the initial (black hole plus no radiation) and final (no black hole plus radiated field quanta) states. This implies the same number of microstates between the initial and final states and thus unitary evolution. This then provides a possible resolution of the information paradox when the specific conditions are imposed.

IV. DISCUSSION AND CONCLUSION

In this article we have used the WKB/tunneling method of calculating Hawking radiation to obtain several non-trivial results concerning the evaporation of black holes and the information paradox taking into account both quantum corrections to all orders in \hbar and/or back reaction. In section II we showed that quantum corrections to all orders in \hbar , as parameterized by the unknown α_j 's, did not alter the spectrum – the spectrum is thermal to all orders in \hbar . Thus, quantum corrections alone can not solve the information loss paradox.

In section III we showed that by taking back reaction and quantum corrections to all orders in \hbar into account *and* assuming some specific form for the quantum corrections (see equation (22)), one could present a possible resolution for the information loss paradox. In this resolution the black hole completely evaporates, but the entropy is conserved. The initial quantum corrected formula for the entropy given in equation (23) is equal to the entropy of the emitted radiation (33). This conservation of entropy indicates that the initial and final states have the same number of microstates and thus that the evolution is unitary.

As a final comment we note that our pragmatic choice of α_j 's in (22) also led to well behaved temperature versus mass behavior for the black hole such as given in figure 2(a) by the all order quantum corrections curve (i.e. the solid curve). “Well behaved” here means that, in contrast to the lowest order Hawking result (i.e. the dashed curve in figure 2(a)) where the temperature diverges as the mass goes to zero, the all order quantum corrections, with our choice of α_j 's, gave a curve which for large mass matched the Hawking

result but at some mass reached a maximum and thereafter decreased to zero as mass went to zero. This “well behaved” behavior is similar to that found using the microcanonical description of Hawking radiation [19] applied to simple black holes or also the lowest order Hawking radiation applied non-commutative black holes [20]. Further in the microcanonical description of reference [19] the total energy (black hole plus radiation) is conserved and the black completely evaporates in a finite time. Similarly, in our analysis of section III we found a conservation of entropy (initial black hole entropy equals the entropy of the emitted radiation). Also the black hole was able to completely evaporate away with the entropy of the initial black hole being converted to entropy of the outgoing radiation. We hope to explore these similarities in the future with an eye toward finding a firmer motivation for our choice of α_j ’s in (22).

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